

DOCUMENT RESUME

ED 374 971

SE 054 924

AUTHOR Sawada, Daiyo
TITLE Mathematics as Problem Solving--A Japanese Way.
PUB DATE Apr 94
NOTE 8p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 4-8, 1994).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Elementary School Students; Elementary School Teachers; Ethnography; Foreign Countries; Grade 5; Intermediate Grades; *Mathematics Instruction; *Problem Solving; *Teaching Methods
IDENTIFIERS *Japan

ABSTRACT

It is known from many international studies that Japanese students do well in mathematics. This paper describes a typical lesson from a Japanese fifth-grade classroom, and discusses the lesson in terms of problem solving, including: embedding of the problem, spending 35 minutes or more on one problem, acting out the problem, multiple solutions, interesting problems to solve, use of manipulative aids, and interpretation rather than application.
(MKR)

* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *

Mathematics as Problem Solving — A Japanese Way

Daiyo Sawada
University of Alberta

A Presentation at the Annual Meeting of the
American Educational Research Association

New Orleans, LA

April 1994

From the NCTM *Standards* (1989:23) we learn that:

The study of mathematics should emphasize problem solving so that students can —

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningfully.

We know from many international studies that Japanese students (and Asian students in general) do quite well in mathematics (see Stevenson & Stigler, 1992 for a rather comprehensive account). One might presume that if mathematics as problem solving is as important as the NCTM would have us believe, then the Japanese must be teaching mathematics as if problem solving were important, even if they are not in any explicit sense "implementing the *Standards*"

What is going on in Japanese elementary school classrooms in regard to mathematics as problem solving? My answer to this question was largely formed during a research visit to the University of Michigan, Ann Arbor at the invitation of Harold W. Stevenson during the Spring/Summer of 1993. His project team was in the midst of analyzing voluminous data gathered from classrooms in Tapei, Beijing, Sendai and Chicago. Dr. Stevenson kindly made it possible for me to immerse myself in a set of classroom observations of 160 lessons taught by 40 teachers of grades 1 and 5 in Sendai, Japan.

The Plan

My plan is not to provide a once over lightly treatment of a few general characteristics of problem solving in Japanese elementary schools. Rather I have

selected a typical lesson at the grade 5 level taught by a typical grade 5 teacher with typically 40 students in her classroom. There aren't many striking characteristics about this teacher that would mark her as exceptional among the set of 40 teachers. I would describe her as competent and careful and caring about her students and about her teaching. That description covers most of the other teachers as well. I present the lesson in some detail to provide the reader with a feel for the problem. I then identify some possible points to notice and comment upon them. In the end the readers needs to come to their own solutions or resolutions. My intent is to provide enough material to make the process worthwhile. I close the article with a question and a partial answer

Carrying Out The Plan

Lesson Description (Grade 5)

The teacher introduces the lesson with a discussion of "crowdedness" [a notion quite meaningful for people of Japan]. After pointing out that one cannot say that a place is crowded simply because it has more people, she focuses discussion onto a particular problem: Which is more crowded, Hokkaido or Okinawa? The teacher reminds the class of problems they have already worked on: "Which room is more crowded, the one that measures 10 mats and has 1 person, or the one that measures 10 mats and has 10 persons? [She is here guiding them to consider a simpler but related problem.] A student volunteers: The first room. The teacher explains that this problem is easy because the rooms are the same size: "But what shall we do since we know that Hokkaido and Okinawa have different areas?" The teacher asks if the two areas can be reduced to the same measurement. A student suggests figuring out how many times bigger Hokkaido is than Okinawa. The teacher explains that this method is similar to the one used in the 'quantity of salt experiment' they worked on earlier, and that this too can be a good method.

Another student suggests that one can "turn over certain area from Hokkaido to Okinawa to make it possible to use calculation." Other students object and the teacher agrees that the method is likely too cumbersome to use here. Another student volunteers that "like what we did with the tatami room (measured in mats), we should try to figure out crowdedness for the same amount of area first (instead of messing around with area, as in the previous suggestion). The teacher applies this suggestion to an example asking, "what then shall be done with the tatami room case?" A student responds, "compare number of persons per mat." The teacher writes this suggestion on the board while repeating it orally.

The teacher returns to a tatami mat problem from a previous lesson and calculates the number of persons per mat for each tatami room and asks which room is more crowded. The teacher confirms the answer by drawing two diagrams on the board showing 1 body and 1/2 a body for room A and 1 body for room B. She elaborates that it is only necessary to calculate "quantity per unit".

Turning to the present problem she asks what unit to adopt. A student responds with "1 square kilometer". Accepting this the teacher points out again that they can base calculation on how many persons per square kilometer. She then summarizes

with: Population divided by area = population per square kilometer and writes this on the board. Students copy this statement into their notebooks. The teacher explains further that by dividing population by area, one figures out how many live in the same amount of space.

In preparation for the calculations, the teacher reminds students about the number of significant digits to keep when rounding. A student responds, "Round off to significant digits when doing calculations" and explains that the numbers given in the problem are already rounded off to 2 significant digits and the teacher confirms. The teacher tells students to solve the problem starting with Hokkaido and helps by having students identify what the rounded numbers are that they should use and writes them on the board. Students work on the calculations at their seats. Teacher circulates. She then asks for an answer and gets "70.8" from a student. She tells him to round it off and then states that Hokkaido has about 71 people per square kilometer.

Students then calculate using the numbers for Okinawa and teacher reminds them that they should use 2 significant digits when doing the calculation and not round off until after they find the answer. A student volunteers "478" and the teacher asks, "Rounding this number to 2 significant digits we have?" A student responds with "480". The teacher then asks which is more crowded, receives the answer that Okinawa is more crowded, and confirms this by reiterating that Hokkaido has 71 people per square kilometer whereas Okinawa has 480 per square kilometer.

Teacher has students look at page 79 in the textbook and asks, "What do we call this 'crowdedness' that we are trying to figure out?" In choral manner students respond, "Population density". The teacher confirms saying, "We have just learnt to figure out population density today." [Another example of embedding a concept in a problem.]

At this point 35 minutes have elapsed. During the remaining 7 minutes students do two more problems involving population density. [Two days later, students were still working on density problems but with some variation: the opening problem involved iron and silver — 250 cc of iron weighing 1975 g; 350 cc of silver weighing 3675 g. This problem took 25 minutes to solve and discuss. The day after this the concept of average was developed as a particular case of density. The problem consisted of comparing the output per factory of two kinds of production where the procedures for handling density generated a quantity which could be called the "average production" (per factory).]

Looking Back

Points of Interest

As an initial reflection, the following points can be listed:

- deliberate effort to connect the present problem to previous problems and solutions,
- the use of simpler problems,
- teacher functions as a guide with a very definite agenda,

- crowdedness provides a meaningful context for embedding the concept of population density,
- in turn, the concept of density provides a meaningful context for embedding the concept of average,
- more than 35 minutes was spent working on this problem,
- student contributions were used to determine the content of the lesson as well as its flow,
- the notion of rounding and significant digits was used consistently when doing the computations,
- the problem was rich in social studies content,
- formulaic statements did not enter the lesson until they could be used as summary confirmations,
- the lesson is not followed up by assigning several "application" exercises; instead two related problems are discussed and solved, each widening the interpretation of the idea of density.

Commentary on the List

Embedding

Problem solving occurs *pervasively* in this lesson. Not only are problem solving techniques used in solving the problem about density, problem solving is also the approach to the lesson structure itself. I have used the term "embedded" as a way to describe this more pervasive use of problem solving in teaching mathematics in which problem solving is the *medium* for mathematics teaching and learning. In the lesson described, there is as much emphasis on the medium (problem solving process) as there is on the message (the particular mathematical content to be learned). This emphasis on the medium in many grade five lessons in Sendai takes precedence over the message so that instructional decisions (how to respond to mathematical "errors" for example) are made with the health of the medium as much in mind as the need to assess the correctness of the message **[need to rework the rest of the paragraph in order to be clear about the relations between solution space and medium]** (for example the so-called error contributes to the richness of the "solution space" and therefore to the health of the medium, and thus is valued even if not particularly "correct" as a message). Indeed, such contributions, because they do enrich the solution space are actively sought through such simple leads as, "Any other methods?" None of these is rejected at the outset and this decision not to reject is based on the health of the medium taking precedence over the correctness of the message. This priority on the medium occurred in the lesson in many particular as well as general ways:

From a mathematical point of view:

- Crowdedness was used as a medium for (or to embed) density,
- Density was used as a medium for Rate/ratio,
- In its turn, Rate/ratio was used as a medium for average.

From a teacher point of view:

- The general problem situation was a medium for the structure of the lesson
- The particular problem was a medium for the mathematical concept

- The multiple solutions generated by students was a medium for the "correct" solution.

From a student point of view:

- The process of solving the problem was a medium for learning of mathematics,
- Right or wrong, their answers defacto were the medium for exploring the solution space,
- Generating and critiquing solutions was the medium for arriving at answers.

Spending 35 minutes or more on one problem

Spending nearly the whole period on one problem might seem to be a very inefficient and drawn out way to teach a mathematical concept. On the other hand if only 4 or 5 minutes were spent on the problem, and given that it really was a problem and not an exercise, then I would have grave doubts that a problem solving approach were being used at all. For a rich problem solving process to take place, there needs to be time for investigating the nature of the problem, generating and proposing multiple solutions, discussing and critiquing various types of solutions, assessing and comparing the relative merits of each solution within the set of solutions, reflecting upon what one has learned and what else could be learned. This sounds like a lot of rhetoric, but it happens on a regular basis in most grade five classes in Sendai (statistically speaking, 86% of Sendai grade 5 teachers use problem solving as a way to embed mathematical concepts, while the comparable figure for Chicago is 14%) and it means that the whole period will often be spent on only one problem. The creative process does not obey a predefined timeline nor do children speak particularly quickly when they are talking about thoughts newly emerging in their minds.

Acting out the problem

Although the students in the lesson described did not act out the solution process, in other Sendai classrooms this happened quite often. The teacher would bring cushions to simulate tatami mats, children would select which one(s) they would prefer to sit on, and a problem of density would be enacted. Rooms with so many mats would be drawn on the board with students selecting which room they preferred and the density problem thus enacted would be ready to be solved.

Multiple Solutions

Multiple solutions are the bread and potatoes (rice and fish) of the problem solving approach used in Sendai. If there was only one solution, it would have to be the correct one so nothing would function as, nor need to function as, a medium for anything else. The correct solution would best be demonstrated or illustrated for all to learn. Multiple solutions, and establishing the conditions for generating, articulating, understanding, critiquing, and comparing multiple solutions, are required components to the way problem solving happens in Sendai classrooms.

Interesting problems to solve

While the problem of the relative population density of Hokkaido and Okinawa may be of interest to Japanese students, as a generalization I must say that most of the problems are rather mundane. Many of them remind me of typical word problems

occurring by the hundreds in American texts. For the classrooms in Sendai, it is not the problems per se that are different; it is the way these problems become the medium of instruction and of learning that is different.

Use of manipulative aids

The NCTM publication *Making the Grade in Mathematics: Elementary School Mathematics in the United States, Taiwan, and Japan* reports a surprising finding: That when manipulatives are introduced into American lessons, the amount of talk decreases while in Japanese elementary classrooms it increases. This finding may have been surprising, but when problem solving is the pervasive mode of teaching and learning, introducing manipulative aids contributes immensely to the variety of multiple solutions to be generated, assessed, compared, etc., thus providing many more opportunities for talk. Thus in a backhanded way, this "surprising" finding is probably just another piece of evidence that problem solving in America exists more in the literature than it does in classrooms.

Interpretation rather than application

Currently in America we talk about developing problem solving strategies and skills and then applying them. In this sense problem solving is split up into two rather distinct parts: (1) the learning of the concepts and skills through problem solving, (2) the use or application of these concepts and skills once learned. This second part is often taken to be the full extent of problem solving: It is for solving problems, not for learning concepts and skills. Our textbooks are organized this way: The concepts are taught (often by demonstration or explanation) and then students are given a collection of similar "problems" to solve. Because of this, classrooms in the U.S. are very likely to end with students doing seatwork. The contrast in Sendai is striking. Classes often end with discussion. And when children are doing seat work at the end of the class it is not only an application of the concepts just learned but often an interpretation of problem situations that extend the ideas beyond the initial circumstances. In a sense the problem solving approach is not two parts but just one. It becomes two parts when the concepts learned (the messages) become so important that they need to be separated out and dealt with differently (as applications). On the other hand if we keep the problem solving process intact and pervasive, the messages learned will never dominate the process that created them.

Conclusion

There are several other aspects about problem solving in Sendai that deserve mention such as the deliberate making of errors (by the teacher) or the occurrence of memorization, but I think the important points have been made. I should like to close this article with a question and a partial answer. Why is it that the problem solving approach as medium has taken hold so pervasively in Japanese elementary schools but remains largely problematic in American schools despite a decade in which nothing in mathematics education received more attention than problem solving ("Let Problem Solving Be The Focus")? While not wishing to oversimplify, I should like to suggest that the notion of a problem solving as a *medium* is as important as the messages it generates, is something that the Japanese are comfortable with. In the West we place such a priority on individuals and on the worth of individuals that no unit of organization could be as important or more important. However, in Japan the

acceptance of Ways of doing things as being as important as the things themselves, is a familiar and comfortable stance. Problem solving as medium is another of these important Ways that allows the nation and all its citizens to participate in the good that life has to offer. But the Way is not routine nor is it easy, despite being within the grasp of everyone. Are we willing to put out the effort so that "Mathematics As Problem Solving" can enter the grasp of everyone?

References

- Becker, J., Silver, E., Kantowski, M.G., Travers, K., & Wilson, J., "Some Observations of Mathematics Teaching in Japanese Elementary and Junior High Schools. (1990). *Arithmetic Teacher* 38 (October, 1990): 12-21.
- Lee, S.Y., Graham, T., & Stevenson, H.W. "Teachers and Teaching: Elementary Schools in Japan and the United States. In T. Rohlen (Ed.) *Teaching in Japan*. Berkeley, CA.: University of California Press, in press.
- National Council Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The Council. 1989
- Stevenson, H.W., Lummis, M., Lee, S., & Stigler, J. *Making the Grade in Mathematics: Elementary School Mathematics in the United States, Taiwan, and Japan*. Reston, VA: National Council Teachers of Mathematics, 1990.
- Stevenson, H.W. & Stigler, J.W. *The Learning Gap: Why Our Schools are Failing and What We Can Learn from Japanese and Chinese Education*. N.Y.: Summit Books, 1992.